MATH 512, SPRING 2022
HOMEWORK 2, DUE MONDAY MARCH 14

Problem 1. *Suppose that $S \subset \omega_1$ is a stationary, co-stationary set. Let $\mathbb{P} = \{c \subset \omega_1 \mid c$ is bounded and closed and $c \cap S = \emptyset\}$, with the order that $c' \leq c$ iff c' end extends c, i.e. for some α, $c' \cap \alpha = c$. Let G be $\mathbb{P}$-generic. Show that in $V[G]$, S is non stationary.*

Problem 2. *Let $j : V \to M$ be an elementary embedding with critical point κ. Suppose that $\mathbb{P}$ is Prikry forcing at κ (for some measure) and let G be $\mathbb{P}$-generic. Recall that in $V[G]$, κ is a singular cardinals with cofinality ω. Show that we cannot lift the embedding j to $V[G]$. In particular, show that any for any generic filter H for $j(\mathbb{P})$, we cannot have that $j``G \subset H$.*

Problem 3. *Suppose that $\mathbb{P}$ is Prikry forcing at κ (for some measure) and let G be $\mathbb{P}$-generic. Show that the set $S = \kappa^+ \cap \mathrm{cof}^V(\kappa)$ does not reflect.*

A **projection** $\pi : \mathbb{P} \to \mathbb{Q}$ is a functions such that:

- $p \leq_{\mathbb{P}} q$, then $\pi(q) \leq_{\mathbb{Q}} \pi(q)$, and
- for every $p \in \mathbb{P}$, for every $q \in \mathbb{Q}$ with $q \leq \pi(p)$, we have that there is a $p' \leq p$, such that $\pi(p') \leq q$.

Problem 4. *Show that if $\pi : \mathbb{P} \to \mathbb{Q}$ is a projection, and G is $\mathbb{P}$-generic, then $H := \{q \in \mathbb{Q} \mid (\exists p \in G)\pi(p) \leq q\}$ is $\mathbb{Q}$-generic.*

Problem 5. *Suppose that $\mathbb{P} * \dot{\mathbb{Q}}$ is a two step iteration.*

(1) *If G is $\mathbb{P}$-generic over V and H is $\dot{\mathbb{Q}}_G$ - generic over $V[G]$, show that $G * H := \{(p, \dot{q}) \mid p \in G, \dot{q}_G \in H\}$ is $\mathbb{P} * \dot{\mathbb{Q}}$-generic over V.*

(2) *Suppose K is $\mathbb{P} * \dot{\mathbb{Q}}$-generic over V and define $G := \{p \in \mathbb{P} \mid (\exists \dot{q})((p, \dot{q}) \in K)\}$ and $H := \{\dot{q}_G \mid (\exists p)((p, \dot{q}) \in K)\}$. Show that G is $\mathbb{P}$-generic over V and H is $\dot{\mathbb{Q}}_G$ - generic over $V[G]$*